Application of Langrangian Theory in Inventory Cost Reduction of a Firm

1 INTRODUCTION

Inventory is classified into the following: Raw Materials, Work in progress, Finished Goods, Components and parts, Maintenance and Repair Supplies and other Operating Supplies. Some of the inventory associated problems include: tie-down on capital, storage limitations, rent on storage, shortages of raw materials supplies, shortages of machine components, shortages of spare parts, obsolescence of inventory items, damage to inventory items, damage to inventory items Pilferage, unpredictable lead times, purchasing problems, among others (Groover, 1987; Adekunle et al., 2018).

Inventory control management can be described as a means of ensuring that the right material, in the right quantity, is made available at the right place at the right time (Tang et al., 2008). Inventories serve to decouple increases the risk of obsolesce. Ploss (1986) observed that inventory is of two types: those concerned primarily with costs and finances which describe inventory as an asset or cash in material form, while those involved with operations define inventory as materials used in the products and as a means of check and balances to run the plant more efficiently by keeping production at a fair level rates and to run reasonably sized manufacturing lots.

Colin (1996) and Oriolowo et al., (2015) described process of controlling inventory as a means of ensuring that stocks are sufficient to meet the requirements of production and sales; also it must avoid holding surplus stock that are necessary and that increases the risk of obsolesce. Ploss (1986) observed that inventory is of two types: those concerned primarily with costs and finances which describe inventory as an asset or cash in material form, while those involved with operations define inventory as materials used in the products and as a means of check and balances to run the plant more efficiently by keeping production at a fair level rates and to run reasonably sized manufacturing lots.

1.1 PARETO ANALYSIS

Pareto Analysis is a formal technique for finding the changes that will give the biggest benefits. Products with low volume have a lower price per unit in compared to those with high volume (Tienza and Zenny, 2017). It is useful where many possible courses of action are competing for an attention. The Analysis serves as a scoping stage of decision making, address and

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prioritizing (Oluwole and Adefemi, 2019).

Pareto Analysis Percentages separated inventory of a firm into three distinct parts, A, B and C - items with specified percentages of separation; A – items correspond to the few quantity of the total items with highest cost out of the total cost of production (Tang et al., 2008). The ABC classification is based on the 70 – 30 principles or Pareto law where about 70% of the total material inventory is represented by 30% of the material inventory. The parameter of this classification is to determine the parameters of Engineering priority, purchasing priority, in which purchasing activities should be focused on the high cost materials and high usage materials and investment decision. Table 1 shows the distributing pattern of the Pareto Analysis.

<table>
<thead>
<tr>
<th>Class</th>
<th>Cumulative % Quantity of Total Item</th>
<th>Cumulative % of Cost of Total Purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0 – 10</td>
<td>0 – 70</td>
</tr>
<tr>
<td>B</td>
<td>11 – 30</td>
<td>71 – 90</td>
</tr>
<tr>
<td>C</td>
<td>31 – 100</td>
<td>91 – 100</td>
</tr>
</tbody>
</table>

Application of this technique allows for concentration of efforts on the area of highest pay-off, which is mostly the A–items (Taha, 2006). Often the amount of space available is restricted by the physical layout of the plant. As a result, the production Manager would like to utilize the space as much as possible.

Maintenance of inventory can be costly through payment of interest, taxes, insurance, and storage rentage. Consequently, there is the need for judicious planning and control of the amount of inventory to keep in order to balance the cost of keeping, replenishment and shortages. The aim of this paper is to design a suitable raw material inventory system for a steel construction firm. The following will be pursued:

(i) Study the present methods of inventory system in the firm.
(ii) Classifying the inventory items found in the firm.
(iii) Study the raw material storage requirement.
(iv) Design and propose appropriate inventory planning and control model best suited for the firm’s operating environment.

1.2 HISTORICAL PERSPECTIVE OF THE STEEL CONSTRUCTION COMPANY

The company’s total premises are 5400m² out of which 3600m² is used as a workshop/storeroom area. The remaining 1800m² consists of Administrative and Executive offices. The company uses 1280m³ of its premises as a raw material inventory system in a crude way. The inventory policy is usually determined by executive opinion, forces of demand and supply which makes their inventory records to be inaccurate. The major raw materials for the company are: 100cm by 100cm angle iron, Gauge 10 Electrode, 70cm by 70cm angle iron, Gauge 12 Electrode and 60cm by 60cm angle iron. 90% of the company’s raw materials were purchased in Lagos, Nigeria on the average of once in 3 months.

2 METHODOLOGY

Various techniques have been proposed by different researchers for analysing inventory systems. The Naddor’s approach with the following four steps was adopted in this study (Tersine, 1981):

(i) Determination of the properties of the system in which the researcher finds out as much as possible about the present inventory situation.
(ii) Formulation of the inventory problem where the researcher identifies the variables that are subject to control.
(iii) Development of a model for the system where the researcher mathematically relates the controllable variables to total cost.
(iv) Derivation of solution in which optimal values of the variables are determined and the optimal cost of the system obtained.

In order to achieve the first step, the following survey techniques were used: interview, record viewing and plant tours. The aim of record viewing was to observe preliminary inventory ordering policies, while plant tour and interviews were carried out to discover the nature of operations and existing material inventory planning and control. To identify the variables that are subject to control, Pareto ABC Analysis was carried out as follows:

(i) List the items, unit costs and their annual usage
(ii) Multiply the unit costs by the annual usage
(iii) Assign a number to rank the items in order, starting with the highest Naira value of annual usage.
(iv) List these items in ranked order and the cumulative annual usage plus the cumulative percentage calculated.
(v) Finally, the items are then classified into A, B and C group by adopting the Pareto Analysis percentages discussed earlier on.

2.1 MODEL DESCRIPTION

The model used in this work considered the inventory system with multiple items which are competing for limited storage and space. This was included in the model as a constraint.

2.2 INVENTORY MODEL

Inventory was categorized along two decision theories, Taha, (2006):

(i) Deterministic models; and
(ii) Probabilistic or stochastic models.

Only one deterministic model was selected for analysis in this research.

2.3 MULTIPLE-ITEM STATIC MODEL WITH STORAGE LIMITATION

The main objectives in using this model are to know:

(i) How much of an item to order within the limits of specified constraints.
(ii) When to place an order for an item.

The following assumptions were made in the model:

(i) Demand was known and constant.
(ii) Replenishment was instantaneous.
(iii) Quantity discount was not allowed.
(iv) Back order was not allowed.
(v) There was a constraint or limitation on the order quantity.
(vi) No shortages were allowed.

2.4 Economic Order Quantity (EOQ) Model
Total cost of inventory, \( C \), is evaluated with equation (1).

\[
C = c_1 I_1 + c_2 I_2 + c_3 I_3
\]

where:
- \( I_1 \) = Inventory Holding Cost,
- \( I_2 \) = Shortage cost, and
- \( I_3 \) = Cost of buying item

Since shortage was not allowed,
\[
C = c_1 I_1 + c_2 I_3 = \frac{c_1 q_i}{2} + \frac{c_2 q_i}{q_i}
\]

Minimize: \( \sum_{i=1}^{N} \left( \frac{c_1 q_i}{2} + \frac{c_2 q_i}{q_i} \right) \)  
(3)

Subject to \( \sum_{i=1}^{N} a_i q_i \leq A \)  
(4)

Total inventory cost for items can be combined:
\[
C(q_i) \text{ where } i = 1, 2, 3, \ldots N
\]

If \( \sum_{i=1}^{N} a_i q_i \leq A \), we place order of \( q_i \)  
(5)

But if \( \sum_{i=1}^{N} a_i q_i > A \), minimize the total cost function subject to the equality constraint. The expression for the multiple-item static model with storage limitation is derived thus (Taha, 2006):

\[
\sum_{i=1}^{N} a_i q_i \leq A
\]

Minimize \( C(q_i, q_2, \ldots, q_N) \),
\[
\sum_{i=1}^{N} \left( \frac{c_1 q_i}{2} + \frac{c_2 q_i}{q_i} \right)
\]

Subject to \( \sum_{i=1}^{N} a_i q_i \leq A, q_i > 0 \) for all \( i \).

The general solution of this problem was obtained by the Langrangian multipliers method.

Let the unconstrained value of \( q_i \) be given as
\[
q_i^* = \frac{2c_2 q_i}{\sqrt{c_1 I_1}}
\]

Satisfies the storage constraint, the constraint is said to be inactive and may be neglected. If the constraint is not satisfied by the values of \( q_i^* \) must be active. In this case, new optimal values of \( q_i \) must be found which satisfy the storage constraint in equal sense.

2.5 Langrangian Function and Multiplier
The Langrangian method is used to modify the objective function being minimized (or maximized) to account for (equality) constraints that restrict the feasible range of the objective function. In other words, this function is used to maximize the utility subject to the budget constraint. Here, cost is the objective function and the storage (space) is the constraint.

By formulating the Langrangian function as:
\[
L(\lambda, q_1, q_2, \ldots, q_N) = C(q_1, q_2, \ldots, q_N) = \lambda \left( \sum_{i=1}^{N} a_i q_i - A \right)
\]

\[
\sum_{i=1}^{N} \left[ \frac{c_1 q_i}{2} + \frac{c_2 q_i}{q_i} \right] - \lambda \left( \sum_{i=1}^{N} a_i q_i - A \right)
\]

(9)

Where, \( \lambda < 0 \) is the Lagrangian multiplier. The optimum values of \( q_i, i = 1, 2 \ldots N \) and \( \lambda \) can be found by equating equation (9) to zero.

Partial derivation of \( L \) with respect to \( q_i \)

\[
\frac{\partial L}{\partial q_i} = \frac{c_1 q_i}{2} + \frac{c_2 q_i}{q_i} - \lambda a_i = 0
\]

for each \( 1, 2, \ldots N \)  
(10)

Secondly, find partial derivatives of \( L \) with respect to \( \lambda \)

\[
\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{N} a_i q_i - A = 0
\]

(11)

Equation (10) implies \( q_i \) must satisfy the storage constraint in equality sense. Solving (10) and (11) simultaneously

\[
q_i^* = \frac{2c_2 q_i}{\sqrt{c_1 I_1}}
\]

(12)

The value \( \lambda^* \) can be found by systematic trial and error. The most negative value of \( \lambda^* \) gives the values of \( q_i^* \) which satisfy the given constraint in equal sense.

2.6 Data Collection
The data was obtained through interview and was used to analyze inventory parameters of the company. 88 items were recorded in which Pareto Analysis was applied which gave 5-items as A-items as shown in Table 2. Ordering cost per order and annual demand for the A – items of the raw materials are shown in Table 3.

<table>
<thead>
<tr>
<th>Table 2. Inventory Items in Class-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item No</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
2.7 Rent Estimation

Building premises cost N15,000,000. The amount was extracted from the company’s record and charged as an annual rental rate. The rent was determined as follows:

PV: Present value = N15,000,000.
SV: Salvage value = 15% of N15,000,000 = N2,250,000.
N: Asset was assumed to have a lifespan of 20 years

Using the straight line method of depreciation in equation (13),

\[ D_n = \frac{PV - SV}{N} \quad (13) \]

\[ D_n = \frac{150,000,000 - 2,250,000}{20} \]

\[ D_n = N637,500 \text{ per annum which is equivalent to the annual rental rate.} \]

Volume of storage room = 16 by 20 by 4m

16 by 20m² of storeroom is equivalent to \( \frac{320}{5400} \times 637,500 = 37,780 \) per annum.

Table 4 shows the ordering cost, annual demand, unit space of raw materials, unit holding cost for each of the A – items of the raw materials.

<table>
<thead>
<tr>
<th>Raw Material</th>
<th>Ordering Cost (c₁)</th>
<th>Annual Demand</th>
<th>Storage space (m²)</th>
<th>Unit space of raw materials</th>
<th>Holding cost for each raw materials</th>
<th>Unit holding cost for each raw materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>100cm by 100cm Angle iron</td>
<td>23700</td>
<td>5000</td>
<td>32.0</td>
<td>0.0064</td>
<td>1150.00</td>
<td>0.23</td>
</tr>
<tr>
<td>Gauge 10 Electrode 70cm by 70cm Angle iron</td>
<td>3350</td>
<td>3000</td>
<td>0</td>
<td>189.5</td>
<td>0.0063</td>
<td>6600.00</td>
</tr>
<tr>
<td>Gauge 12 Electrode 60cm by 60cm Angle iron</td>
<td>14700</td>
<td>1100</td>
<td>0</td>
<td>69.0</td>
<td>0.0063</td>
<td>2420.00</td>
</tr>
<tr>
<td>Gauge 12 Electrode 60cm by 60cm Angle iron</td>
<td>22500</td>
<td>2500</td>
<td>0</td>
<td>157.5</td>
<td>0.0063</td>
<td>5500.00</td>
</tr>
</tbody>
</table>

2.8 Inventory Model Using Langrange Multiplier

\[ q = \frac{-25\gamma T \pi}{c_1 + c_2 + c_3} \quad (14) \]

where total available storage volume is given by \( A = 1280m^3 \) using equation (14), Langrangean iterations below was obtained as in Table 5.

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( q_4 )</th>
<th>( q_5 )</th>
<th>( \sum q_i q_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6420</td>
<td>6045</td>
<td>7686</td>
<td>14302</td>
<td>6281</td>
<td>+1.2</td>
</tr>
<tr>
<td>-0.1</td>
<td>6402</td>
<td>6028</td>
<td>7646</td>
<td>14261</td>
<td>6263</td>
<td>+0.8</td>
</tr>
<tr>
<td>-0.2</td>
<td>6385</td>
<td>6011</td>
<td>7625</td>
<td>14221</td>
<td>6246</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

*optimal quantity

The storage constraint is satisfied in equality sense for some value of \( \lambda \) between –0.1 and –0.2. This value is equal to \( \lambda^* \) and may be estimated by linear interpolation. The corresponding values of \( q_i \) should thus yield \( q_i^* \) directly. Since from the table \( \lambda^* \) appears remarkably close to –0.2, optimal \( q_i^* \) are given in Table 5.

2.9 Analysis of the Company’s Inventory for Raw Materials

The present inventory policy of the company shows a quarterly ordering for raw materials.

Inventory Cost \( = c_{31} \left( \frac{D}{q} \right) + c_{11} \left( \frac{Q}{2} \right) = 4c_{31} + c_{11} \left( \frac{D}{B} \right) \) \( (15) \)

\( Bi – annual inventory policy is obtained as 2c_{31} + c_{11} \left( \frac{D}{4} \right) \)

\( Annual inventory policy is obtained as c_{31} + c_{11} \left( \frac{D}{2} \right) \)

Annual inventory cost for an Economic order quantity can be computed as

\[ c_{31} \left( \frac{Q}{q} \right) + c_{11} \left( \frac{Q}{2} \right) \quad (16) \]

After exploring some inventory policies as stated in (15), summary of the Annual Total Inventory cost for each of the policies are shown in Table 6.

<table>
<thead>
<tr>
<th>Inventory Policy</th>
<th>Annual Total Inventory Cost (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly</td>
<td>307,446.25</td>
</tr>
<tr>
<td>Bi-annual</td>
<td>154,667.50</td>
</tr>
<tr>
<td>Annual</td>
<td>84,035.00</td>
</tr>
<tr>
<td>Annual Economic Order</td>
<td>45,110.65</td>
</tr>
<tr>
<td>Quantity (Proposed)</td>
<td></td>
</tr>
</tbody>
</table>

3 Results and Discussion

The annual data of the company was used for the analysis. The rent constitutes 87% of the estimated total holding costs. The storage capacity for raw materials was estimated at 1280m³. By considering the most “vital few” of the raw materials (A – items): 100cm by 100cm Angle iron, Gauge 10 Electrode, 70cm by 70cm Angle iron, Gauge 12 Electrode, 60cm by 60cm Angle iron constitute 2.5%, 14.8%, 5.4%, 12.3%, 4.9% of total space capacity.
respectively. From Table 6, quarterly inventory policy cost is $307,446.25 per annum. The ordering cost constitutes 97.7% of the total inventory cost while holding cost constitutes 2.3% of the total inventory cost. Bi-annual inventory policy was $154,667.50 per annum, the ordering cost was 97.1% of the total inventory cost while the holding cost is 2.9% of the total inventory cost.

For the Annual policy, the total inventory cost was $84,035.00 per annum. The ordering cost was estimated as 89.4% of the total inventory cost while the holding cost was also estimated at 10.6% of the total inventory cost. The annual cost of inventory for an Economic Order Quantity (proposed model) was put at $45,110.65 per annum. The ordering cost was estimated as 50.3%, and holding cost was estimated as 49.7% of the total inventory cost. If the company adopts Annual Policy instead of Quarterly policy which has been its tradition, sum of $22,341.25 would be saved and more so, if the Annual Economic Order Quantity (proposed model) is adopted, the sum of $62,335.60 would be saved. From the research work, it was discovered that the model described would be feasible with the company’s inventory system. The survey showed that a minimum of one week is required to order and receive raw materials.

4 CONCLUSIONS
It was observed that the company adopted no scientific inventory control procedures. The results of this work showed that:
(i) The model used is feasible with the inventory system of the company.
(ii) The present quarterly ordering policy is not the best in the circumstances.
(iii) An annual ordering policy is to be preferred. This leads to a savings of N223,411.25 per annum which is estimated to be 72.7% reduction in inventory cost per annum over the present policy.
(iv) If the Economic Order Quantity (proposed model) is adopted, company will save N62,335.60 (85%) of the inventory cost per annum over the present policy.

NOTATIONS:
C Total Cost of Inventory,
Ii Inventory Holding Cost,
Is Shortage Cost,
Ib Cost of buying item,
qi Optimal quantity of item i,
a Unit space occupied by an item i,
A Total cubic metre of storage space,
ci Holding cost for item i,
ci Ordering cost for item i,
λ Langrangian multiplier,
R Annual Demand,
Q Order quantity,
D Demand,
q* Optimal quantity.

REFERENCES


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