Capacity Analysis of a Full Duplex Device-to-Device Wireless Network using Voronoi diagrams and Distance Distributions

Okechukwu E. Ochia and *Elvis Obi

Department of Communications Engineering, Ahmadu Bello University, Zaria, Nigeria
{emmiralde/elvisobi}@gmail.com

Abstract— Full duplex (FD) and Device-to-Device (D2D) communication are two revolutionary protocols that have enabled better spectrum utilization and more reliable data delivery in wireless networks. In addition, stochastic geometry tools have become necessary to characterize the randomness in the present networks with respect to the irregular architecture and the competing access schemes. This work analyses the performance of a mobile network comprising nodes which are randomly distributed in a square area, which are equipped with FD radios, and can communicate using D2D. The base station (BS) nodes and user nodes in the network are modelled as points of a homogeneous binomial point process (BPP) and a homogeneous Poisson point process (PPP) respectively. The network area is tessellated into cells using Voronoi diagrams which approximates to a nearest BS-to-user node association policy. The user nodes can cache popular file objects which are available in a centralized server in the network and other nodes in proximity.

Keywords — Device-to-Device Communication, Full Duplex, Stochastic geometry analysis, Voronoi diagrams, Distance Distributions

1 INTRODUCTION

The contemporary wireless networks such as mobile ad-hoc networks (MANETS), wireless sensor networks (WSNs), cognitive radio networks (CRNs) and heterogeneous cellular networks (HCNs) are becoming more complex, with different device types, multiple coexisting tiers and competing communication protocols. Since the seminal work of Gupta and Kumar (2000) where the capacity of a wireless network comprising an arbitrary distribution of nodes was analyzed using a protocol and physical model, research has been geared towards investigating performance of networks with non-deterministic node locations and irregular traffic patterns. In a separate stream, stochastic geometry has shown to provide a useful set of tools for analyzing the performance of interference-limited networks where classical methods of communication theory have become insufficient (Haenggi et al., 2009).

The classical grid-based model has been used in network planning and research where inter-site distances are assumed to be deterministic, and coverage patterns, uniform. However, there is an increasing variance between grid-based models and actual real-world deployments due to randomness in spatial distances (Oh & Krishnamachari, 2010). Furthermore, it is impossible for a wireless node to predict locations of all but a few other nodes in the network for estimating target and interfering channels for signal processing or interference cancellation (Baccelli & Blaszczyszyn, 2009). Point process theory, a sub-field of stochastic geometry, has emerged as an alternative for capturing randomness in non-deterministic wireless networks. The PPP-based model is shown to be analytically tractable and also compliant with actual fourth-generation cell deployments and better captures increasingly opportunistic and dense placement of base stations (BSs) in present heterogeneous cellular networks having multiple tiers (Andrews et al., 2011).

D2D communication enables the direct exchange of data between mobile nodes without the need for routing through the BS. It leverages the increasing size and low-cost storage available on mobile terminals which enables them to selectively cache contents of interest from the cloud or other network server and exchange content of interest with other nodes. D2D communications has also been proposed in the Long Term Evolution-Advanced (LTE-A) specifications for increasing user demand and provide better user experience from a spectral efficiency (SE), throughput and delay perspective and also finds applications in other types of cache-enabled wireless networks (Feng et al., 2013). With advances in analog and digital successive interference cancellation (SIC) techniques, devices having transceivers operating in FD can simultaneously transmit and receive on the same frequency channel, thus enabling spectrum needs to be cut by up to fifty percent to realize the same quality of service target. With adequate SIC, FD has been shown to provide significant gains in sum throughput and a reduction in content download latency compared to the half-duplex (HD) that requires a time division duplex (TDD) or frequency division duplex (FDD) scheduling scheme (Nascherga et al., 2017).

This work analyses the capacity of a FD-D2D wireless network using Voronoi diagrams and distance distributions. The network is assumed to be of square size and comprises a BS tier and a D2D tier. The BS nodes are drawn from a homogenous BPP with a specified intensity and the user nodes are modelled as points of a stationary PPP with a different intensity greater than the BS intensity. The Voronoi tessellation in space is used to subdivide the network into irregular polygons or cells with each cell seed corresponding to a BS and its polygon corresponding to its coverage area. The users can randomly cache content retrievable from a centralized server and under a BS coordination scheme. For the sake of simplicity, it is assumed that users can only cache one file at a time and all files have the same size. Users can request for other files not in their cache from other users and the request statistics are modelled...
using the Zipf distribution with a skew parameter. It is assumed that D2D communications is scheduled as downlink transmissions with a bandwidth allocation while uplink analysis is deferred to future work.

All D2D transmissions take place concurrently; hence a user of interest experiences interference from all other transmissions. Following a protocol model, a D2D link can be established between a pair of users if their pairwise Euclidean distance is below a distance threshold. Further, the coverage probability is characterized using a physical model where the received signal to interference and noise ratio (SINR) at a user should be above a specified threshold for successful decoding of received content. More so, FD links can be established when transmitter-receiver pairs that have cached and requested files respectively also satisfy the distance based and SINR based thresholds. Following the assumptions in the design, the cumulative distribution function (CDF) of the pairwise distances between users are characterized using expressions for distributions in random networks according to results in (Molchanov, 2012). The D2D communication probability based on the distance threshold is computed analytically and the result is validated using simulations. Analytical expressions for the FD and coverage probabilities are derived and simulations are used to compute the sum and average throughput.

In (Bharadia et al., 2013), It was shown that an in-band FD WiFi radio can achieve close to doubling the theoretical throughput compared to a half-duplex (HD) system with the same frequency resources. In addition, the authors in (Wen et al., 2017) presented a framework for analyzing the spectral efficiency (SE) and energy efficiency (EE) tradeoff in a cellular network with FD enabled BS. Considering a practical multi-user network with residual self-interference due to imperfect FD radios, a closed form quasi-concave SE-EE expression was derived. A heuristic algorithm based on a Lagrange dual decomposition was proposed for maximizing the SE-EE performance. Dozens of works analyzing the performance of D2D communication in different cellular architectures have equally been presented in the literature. In (Golrezaei et al., 2013), a new network architecture was proposed which combines the notion of content caching in femto-base stations and distribution via helper nodes using D2D communications. The authors showed that for scenarios where users request for few but highly popular video files, the system throughput is significantly improved when the files are biased to be located close to the requesting users and are delivered via D2D. The promised gains realizable from using D2D have also been extended to other system architectures such as the more recent cloud radio access network (C-RAN). In (Liu et al., 2016), D2D was integrated into a C-RAN with coordinated multipoint and results showed that by properly designing the D2D protocol, the additional cross-tier interference introduced by the D2D links were offset by the significant gains in the average spectral efficiency (ASE) of the network.

The usefulness of stochastic geometry and spatial modelling in analyzing performance of wireless networks was laid out in (Andrews et al., 2010) where the authors discussed tradeoffs between the accuracy of the selected point processes for modeling real network deployments and the loss in analytical tractability. Thus contributions of this work are four-fold:

- A spatial network model for characterizing the locations of the nodes based on the binomial and Poisson point processes is proposed.
- Different from existing works, a user-to-BS association policy based on a Voronoi tessellation of the network area is presented.
- A D2D link establishment protocol parameterized by the inter-node distances is proposed and the D2D coverage area is modelled using a thinning point process.
- The expressions for the coverage probability, collaboration probabilities and the achievable rate for a finite network dimension are derived with analysis and insights provided to maximize the system performance.

2 SYSTEM DESIGN AND METHODOLOGY

In this section, a background is given on the design aspects of the system. The spatial distribution of the BS and user nodes is described in 2.1. The D2D protocol model as well as the content caching policy is discussed in 2.2. Based on a physical model and stochastic geometry analysis, analytical expressions for the downlink or D2D coverage and FD collaboration probabilities are provided in 2.3

2.1 SPATIAL DISTRIBUTION OF NETWORK NODES AND USER ASSOCIATION

The network is assumed to be of square size with a unit area. The BS nodes in the network area are drawn from a homogeneous PPP $\Phi_{BS}$ with arbitrary intensity $\lambda_{BS}$. The users are drawn from a stationary PPP $\Phi_u$ with an intensity, $\lambda_u$ which is an order of magnitude greater than $\lambda_{BS}$. The network is subdivided into Voronoi cells using a Voronoi tessellation in space. The Voronoi cell, $V(x)$ of a point (x) of a general point process $\Phi \subset \mathbb{R}^d$ consists of those locations of $\mathbb{R}^d$ whose distance to x is not greater than its distance to any other point in $\Phi$ (Haenggi et al., 2009) i.e.

$$V(x) \triangleq \{ y \in \mathbb{R}^d : \|x - y\| \leq \|z - y\| \quad \forall z \in \Phi(x) \} \quad (1)$$

Where $\|\cdot\|$ is the Euclidean distance metric.

The Voronoi tessellation is a decomposition of the space into the Voronoi cells of a general point process. This decomposition is applied to the network model where each point x is a BS node and its corresponding $V(x)$ is an irregular polygon that represents its coverage area. $z$ and $y$ represent other BS and user nodes in the network respectively. Figure 1 illustrates a snapshot of the Voronoi tessellation of the network with a $\lambda_{BS}$ of 10 nodes/square units and a $\lambda_u$ of 100 nodes/ square units i.e. a relative density $\lambda_u/\lambda_{BS}$ of 10.
2.2 D2D LINK ESTABLISHMENT PROTOCOL

All user nodes are assumed to have a single transceiver antenna and can engage in one hop D2D transmissions and receptions based on a protocol model. In this model, the Euclidean distance between any two users is the objective metric. If this distance is less than or equal to a specified distance threshold, a D2D link can be established between the pair of users and no link is established if otherwise. This represents a thinning process on \( \Phi_U \). The link establishment protocol for a pair of D2D users is thus given as:

\[
u_i, u_j \in \Phi_{D2D} \leftrightarrow r_{ij} \triangleq \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \leq r_{D2D}
\]

(2)

Where,

\( u_i, u_j \) is an arbitrary pair of users located in the network \( \Phi_{D2D} \) is the D2D point process \( x_{i,j}, y_{i,j} \) are the x and y coordinates of users \( u_i, u_j \)

Based on results in (Moltchanov, 2012) and earlier analysis in (Miller, 2001), the cumulative distribution function (cdf) for the pairwise distance between points that are uniformly distributed in a 2-dimensional square area is given by:

\[
F_r(v = r_T D) = \begin{cases} 
0 & v < 0 \\
\frac{\sqrt{u^2 + \frac{1}{2} v^2} - \frac{8}{3} u^2 + \pi}{4 - v^2} & 0 \leq u \leq 1 \\
\frac{2}{3} v^2 \left(\frac{\sin^{-1} \frac{1}{u}}{u} - \frac{1}{2} \right) - \frac{1}{3} v^4 + \frac{2}{3} v^2 - \frac{1}{3} & 1 \leq u < \sqrt{2} \\
\end{cases}
\]

Finally, \( F_r(v = r_T D) = 1, \forall \, v \geq \sqrt{2} \)

Where,

\( F_r(\cdot) \) is the cdf of the distance
\( \gamma \) is the distance threshold
\( D \) is the size of the square area
\( r_T \) is the normalized distance threshold

\( v \) is a parameter that defines the boundaries of a unit square.

Hence, the D2D link establishment probability, \( P_{D2D} \) is defined as:

\[
P_{D2D} \equiv \mathbb{P}(r_{ij} \leq r_{D2D}) = F(r_{D2D} = r_T D)
\]

(4)

The D2D link protocol corresponds to an independent thinning transformation in \( \Phi_U \), and hence \( \Phi_{D2D} \) has an intensity, \( \lambda_{D2D} \), given as:

\[
\lambda_{D2D} = \frac{\lambda_U}{\lambda_{D2D}}
\]

(5)

2.3. CONTENT CACHING, FD COLLABORATION, AND COVERAGE PROBABILITY

Under a BS coordination scheme, each \( u_i \) is assigned to cache a single file, \( f_i \) according to a random independent caching scheme. A user \( u_k \in \Phi_{D2D} \) requests for a file not in its cache, according to a Zipf distribution with a skew exponent, \( \nu_r \). The probability of requesting for a file \( f_i \) is given as:

\[
P(f_i) = \frac{i^{-\nu_r}}{\sum_{j=1}^{N_i} j^{-\nu_r}}
\]

(6)

Where \( M \) the cardinality of the ordered is set, \( F \) of popular files, available at the centralized server and \( i,j \) represent the ranked indices of \( f_i \) and other files \( f_j \) respectively.

The BS has full knowledge of the cached content in each user as well as their locations and requests. Once all D2D users have been identified, the BS initiates a broadcast session, informing all potential D2D transmitters, \( T \subset \Phi_{D2D} \) with the requested content in their cache to broadcast pilot symbols to all other users. The corresponding D2D receivers, \( R \subset \Phi_{D2D} \) will proceed to receive their requested content from the transmitters based on a physical model. It is worth noting that some users will be transmitters only, others will be both transmitters and receivers while others may remain idle. The D2D transmit and receive probabilities are analyzed using similar analysis in (Naslecheraghi et al., 2017). A D2D user is a receiver if its requested file \( f_i \) is not found in its cache and is available in the virtual cache, i.e.

\[
\begin{align}
R_x & = \Pr(u_i \in R \cap f_i \notin F_{D2D} \cap f_i \notin u_i) \quad (7a) \\
& = \Pr(u_i \in R) \cdot \Pr(f_i \in F_{D2D}) \cdot \Pr(f_i \notin u_i) \\
& = \frac{N_R}{N_{\Phi_{D2D}}} \cdot \left( \frac{\Sigma_{i=1}^{N_{\Phi_{D2D}}} i^{-\nu_r}}{\Sigma_{j=1}^{N_i} j^{-\nu_r}} \right) \left( 1 - \frac{1}{N_R} \right) \\
& = \frac{N_R}{N_{\Phi_{D2D}}} \cdot \left( \frac{\Sigma_{i=1}^{N_{\Phi_{D2D}}} i^{-\nu_r}}{\Sigma_{j=1}^{N_i} j^{-\nu_r}} \right) \left( 1 - \frac{1}{N_R} \right) \quad (7c)
\end{align}
\]

(3)

Where (7a) and (7b) follow from the joint independence property of the three events.

\( R \) is the set of D2D receivers, \( N_R \) and \( N_{\Phi_{D2D}} \) are the cardinalities of \( R \) and \( \Phi_{D2D} \) respectively, \( F_{D2D} \) is the set of files available in the virtual D2D cache and \( M \) is the cardinality of the available popular files. Since all users are identically and independently distributed, \( \frac{N_R}{N_{\Phi_{D2D}}} \) is the probability that a D2D node is a receiver; \( \Pr(f_i \in F_{D2D}) \) is the probability that \( f_i \) has an index smaller than or equal to \( N_{\Phi_{D2D}} \) and \( \Pr(f_i \notin u_i) \) is the probability that \( f_i \),
is not cached in \( u_i \) given that a single file is cached per user node.

A node, \( u_i \) acts as a transmitter if at least one node other than it, requests its cached file \( f_i \). Therefore, the probability of being a D2D transmitter is given by:

\[
P_{tx} = Pr(f_i \in F_t \cup Q_i) \left( Pr(u_i \in R) \right) + Pr(f_i \in F_d) \left( Pr(u_i \notin R) \right)
\]

\[
= \sum_{i=1}^{N_{\Phi D2D}} \left( 1 - \left( 1 - P(f_i) \right)^{N_t} \right) \frac{N_g}{N_{\Phi D2D}} \frac{1}{N_T} + \sum_{i=1}^{N_{\Phi D2D}} \left( 1 - \left( 1 - P(f_i) \right)^{N_t-1} \right) \frac{1}{N_T}
\]  

(8a)

Where \( F_t \) is the set of files requested by the users, \( Q_i \) is the request of the \( i \)th user, \( N_t \) is the cardinality of \( T \) and the expectation is taken over all the cached files in \( \Phi D2D \).

A device communicates in FD mode when it acts as a receiver and a transmitter. Since the transmit and receive events are independent, the probability of operating in FD mode is given by:

\[
P_{FD} = P_{tx} \cdot P_{rx}
\]  

(9)

The SINR \( \beta \) of a typical D2D receiver with analysis permissible for a homogeneous PPP by Slivnyak’s theorem is given by:

\[
\beta = \frac{P_t h_{rx} r^{-\alpha}}{N_o + I_{agg} + \psi P_t}
\]  

(10)

Where \( P_t \) is the fixed transmit power of the D2D pairs, \( h_{rx} \sim \exp(1) \) is the exponential random channel coefficient with unit mean power, \( \alpha \) is the path loss exponent, \( N_o \) is the additive white gaussian noise at the receiver with a specified variance \( \sigma^2 \), \( I_{agg} \) is the aggregate interference received from other D2D transmissions since they all use the same channel resource and \( \psi \) is the residual fraction of self-interfering power after SI due to FD communication occurring on the same frequency. The aggregate interference power at a typical user is defined as:

\[
I_{agg} = I_{inter} + I_{intra}
\]

\[
= \sum_{z \in \mathcal{V}_{\Phi D2D}} P_t h_{zo} r_{zo}^{-\alpha} + \sum_{y \in \mathcal{V}_{\Phi D2D}} P_t h_{yo} r_{yo}^{-\alpha}
\]  

(11a)

(11b)

Where the first summation in (11b) is the inter-cellular interference from the D2D nodes in other cells beside the target user’s cell, while the second term is the intra-cellular interference from the D2D nodes in the same cell with the target user. \( h_{zo} \) represents the random channel attenuation on the link between the interferers and target user.

Based on a physical model and following similar analysis in (Shi et al., 2009), a D2D node pair, \( u_i, u_j \) a distance \( r \) apart can engage in successful communication if the SINR at the receiver node is greater than an arbitrary threshold. Hence the downlink coverage probability or SINR complementary cumulative distribution function (ccdf) averaged over the pairwise distances, \( r_{ij} \) is

\[
P_{cov}(\beta, \alpha, D) \triangleq \mathbb{E}_r \left[ Pr(\beta > \beta_T | r) \right]
\]

\[
= \int_{r_{min}}^{r_{max}} P \left( h_r > \frac{\beta r^{-\alpha} (N_o + I_{agg} + \psi P_t)}{P_t} \right) f_r(r) dr
\]

(12a)

\[
= \int_{r_{min}}^{r_{max}} \exp \left( -\frac{\beta r^{-\alpha} (N_o + \psi P_t)}{P_t} \right) E \left[ \exp \left( -\frac{\beta r^{-\alpha} I_{agg}}{P_t} \right) \right] f_r(r) dr
\]

(12b)

\[
= \int_{r_{min}}^{r_{max}} \exp \left( -\frac{\beta r^{-\alpha} (N_o + \psi P_t)}{P_t} \right) \mathcal{L}_{I_{agg}} \left( \frac{\beta r^{-\alpha}}{P_t} \right) f_r(r) dr
\]

(12c)

Where the integrand in (12b) follows from substitution and simple manipulation of (12a) and subsequent integration over the distribution of \( r \). (12c) stems from the fact that \( h_r \sim \exp(1) \) with the second expectation term representing the Laplace transform, \( \mathcal{L}_X(s) \) in (12d) of a random variable, \( X \) where \( X = I_{agg} \) and the parameter \( s \leq \frac{\beta r^{-\alpha}}{P_t} \).

\( f_r(r) \) is the probability density function (pdf) of the pairwise distance metric and is related to its piece-wise ccdf by:

\[
f_r(r) = \frac{dF_r(r)}{dr}
\]  

(12e)

Following the analysis for the coverage probability, the achievable throughput for a D2D link, \( l \) with SINR, \( \beta \) and threshold SINR, \( \beta_T \) is given by:

\[
T_l = \log_2(1 + \beta) Pr(\beta > \beta_T)
\]  

(13)

By averaging over all D2D links in coverage, the average throughput for a typical receiver is:

\[
T_{avg} = \log_2(1 + \beta) P_{cov}
\]  

(14)

Finally, the mean rate per D2D user is given as:

\[
R_{D2D} = \frac{T_{avg}}{\lambda_{D2D} * D^2}
\]  

(15)

Where \( D \) is the dimension of the square network.

In the subsequent analysis, the assumed parameters are as follows: distance threshold \( r_v = 0.2, 0.5, 0.8 \); popularity exponent \( v_r = 0.5, 1.0, 1.5 \); transmit power, \( P_t = 1 \); self-interference factor, \( \psi = 0 \) and noise variance \( N_o = 0 \). In all cases, all investigated metrics were computed via Monte-Carlo simulations by averaging over \( 10^4 \) runs and the experimental results were validated with theory.

3 Results and Discussions

In section 3.1, the D2D link establishment and FD collaboration probabilities for varying distance thresholds, relative node densities and skew file request exponents are plotted using theoretical results and verified with simulations. The variation in the SINR ccdf or coverage probability as a function of relative node densities is illustrated in 3.2. Finally, 3.3 illustrates the scaling behavior of the sum throughput with the network density.
3.1 D2D Link Establishment and FD Collaboration Probability

The variation of the D2D link establishment and FD collaboration probabilities with the D2D threshold distance are presented in Figure 2 and 3 respectively. The analytical results are closely matched with simulations as illustrated in the figures. In Figure 2, the fraction of D2D links is seen to increase with the threshold distance since with higher threshold distances, more nodes that are further apart in the network are permitted to establish D2D sessions.

Figure 3 illustrates the independence between the D2D probability and the node density and alludes to the fact that the link establishment protocol is independent of the network size and only dependent on the network geometry and dimensions.

Lastly, Figure 4 illustrates the behavior of the FD collaboration probability with the D2D node density for different request exponents and for a chosen distance threshold. In all request instances, the collaboration probability increases with the node density due to increasing fraction of requests for available files thus enabling more FD communication between transmitters having the requested files in their cache. For higher exponent values, there is increased redundancy in the requests meaning that fewer files account for the requests. Therefore, a single transmitter having a requested file in its cache can broadcast the file to two or more nodes having identical requests, thus leading to a higher likelihood of FD communication.

3.2 D2D Coverage Probability

The coverage decreases for increasing threshold SINR in Fig. 5 as a lesser proportion of D2D receptions have the required SINR to meet the desired threshold for successful reception when the threshold is increased.

3.3 Mean Rate per D2D User and Average Throughput

In both Figs. 6 and 7, the throughput and mean rate (which is average throughput per user) decreases with increasing node density due to increase in the interference power coming from the higher number of concurrent transmissions occurring with the target transmission. For a higher skew exponent, there are fewer unique file requests leading to more multicast
transmissions of identical files and hence reduced interference to the target receiver.

4 CONCLUSIONS
The results of this work are useful in understanding the capacity bounds in FD enabled networks from a design and QoS perspective. They also provide an analytical framework enabling extensions to uplink (user to BS) performance and provides a baseline for performance comparison with HD only networks.

REFERENCES


